DAY THIRTY TWO

Atoms

Learning & Revision for the Day

- Scattering of α-particles
- Rutherford's Model of an Atom
- Bohr's Model
- Energy Levels and Hydrogen Spectrum

Atom is the smallest particle of an element which contains all properties of element. Molecule is a single atom or a group of atoms joined by chemical bonds. It is the smallest unit of a chemical compound that can have an independent existence. Nuclei refers to a nucleus of an atom, having a given number of nucleons. It is a general term referring to all known isotopes–both stable and unstable of the chemical elements. Thus, O^{16} and $O^{17}$are different nuclides.

Scattering of α-particles

In 1911, Rutherford successfully explained the scattering of α-particles on the basis of nuclear model of the atom.

Number of α-particles scattered through angle θ is given by

$$N(\theta) \propto \frac{Z^2}{\sin^4(\theta/2)\,K^2}$$

where, K is the kinetic energy of α-particle and Z is the atomic number of the metal.

At distance of closest approach the entire initial kinetic energy is converted into potential energy, so

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze(2e)}{r_0}$$

$$\Rightarrow \qquad r_0 = \frac{Ze^2}{mv^2\pi\varepsilon_0} = \frac{4KZe^2}{mv^2}$$

Rutherford's Model of an Atom

On the basis of scattering of α-particles, Rutherford postulated the following model of the atom

- Atom is a sphere of diameter about 10^{-10} m. Whole of its positive charge and most of its mass is concentrated in the central part called the nucleus.
- The diameter of the nucleus is of the order of 10^{-5} m.
- The space around the nucleus is virtually empty with electrons revolving around the nucleus in the same way as the planets revolve around the sun.
- The electrostatic attraction of the nucleus provides centripetal force to the orbiting electrons.

- Total positive charge in the nucleus is equal to the total negative charge of the orbiting electrons.
- Rutherford's model suffers from the following drawbacks
  (a) stability of the atomic model.
  (b) nature of energy spectrum.

Bohr's Model

Bohr added the following postulates to the Rutherford's model of the atom

- The electrons revolve around the nucleus only in certain permitted orbits, in which the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is the Planck's constant

$$\left(L = mv_n r_n = \frac{nh}{2\pi}\right)$$

- The electrons do not radiate energy while revolving in the permitted orbits. That is, the permitted orbits are stationary, non-radiating orbits.
- The energy is radiated only when the electron jumps from an outer permitted orbit to some inner permitted orbit. (Absorption of energy makes the electron jump from inner orbit to outer orbit).
- If energy of the electron in nth and mth orbits be E_n and E_m respectively, and when the electron jumps from nth to mth orbit the radiation frequency ν is emitted, such that $E_n - E_m = h\nu$. This is called the **Bohr's frequency equation**.

NOTE
- Radius of the orbit of electron in a hydrogen atom in its stable state, corresponding to $n = 1$, is called Bohr's radius. Value of Bohr's radius is $r_0 = 0.529\ \text{Å} \approx 0.53\ \text{Å}$.
- The time period of an electron in orbital motion in the Bohr's orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.53\text{Å}}{\frac{c}{137}} = 1.52 \times 10^{-6}\ \text{s} \quad \left[\because v = \frac{c}{137}\right]$$

and the frequency of revolution is $f = \frac{1}{T} = 6.5757 \times 10^{15}$ cps

Some Characteristics of an Atom

- The **orbital radius** of an electron is

$$r_n = 4\pi\varepsilon_0 \frac{n^2 h^2}{4\pi^2 Zme^2} = 0.53\frac{n^2}{Z}\ \text{Å}$$

- The **orbital velocity** of an electron is

$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{2Z\pi e^2}{nh} = \left(\frac{c}{137}\right)\frac{Z}{n} = 2.2 \times 10^6 \left(\frac{Z}{n}\right)\ \text{m/s}$$

- **Orbital frequency** is given by

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

- The **total energy** of the orbital electron is

$$E = -\left(\frac{me^4 Z^2}{8\varepsilon_0^2 h^2 n^2}\right)$$
$$= -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right) ch \frac{Z^2}{n^2}$$
$$= -Rch\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}\ \text{eV}$$
$$\text{KE} = \frac{me^4 Z^2}{8n^2 h^2 \varepsilon_0^2}, \quad \text{PE} = -\frac{me^4 Z^2}{4n^2 h^2 \varepsilon_0^2}$$

- The kinetic, potential and total energies of the electron with r as the radius of the orbit are as follows

$$\text{KE} = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right]$$
$$\text{PE} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

and
$$E = -\frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right]$$

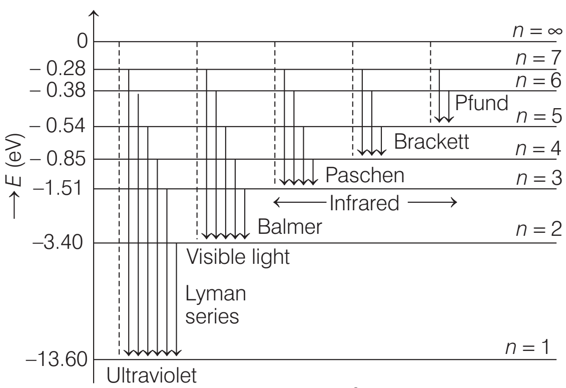
Therefore, they are related to each other as follows

$$\text{KE} = -E \text{ and } \text{PE} = 2E$$

- For a hydrogen atom, $r_n \propto n^2$, $v_n \propto \frac{1}{n}$ and $|E| \propto \frac{1}{n^2}$
- The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is $\Delta L = (n+1)\frac{h}{2\pi} - \frac{nh}{2\pi} = \frac{h}{2\pi}$

Energy Levels and Hydrogen Spectrum

Hydrogen spectrum consists of spectral lines classified as five spectral series of hydrogen atom. Out of these five, Lyman series lies in the ultraviolet region of spectrum, Balmer series lies in the visible region and the remaining three series, lie in the infrared region of spectrum.



Emission spectrum of H-atom

Total number of emission spectral lines from some excited state n, to another energy $n_2 (< n_1)$ is given by

$$\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$$

e.g. total number of lines from $n_1 = n$ to $n_2 = 1$ are $\frac{n(n-1)}{2}$.

The five spectral series of hydrogen atom are given below

1. Lyman Series

Spectral lines of Lyman series correspond to the transition of electron from higher energy levels (orbits) $n_i = 2, 3, 4, \ldots$ to ground energy level (1st orbit) $n_f = 1$.

For Lyman series, $\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]$

where, $n = 2, 3, 4, \ldots$

It is found that a term $Rch = 13.6 \text{ eV} = 2.17 \times 10^{-18}$ J. The term Rch is known as Rydberg's energy.

2. Balmer Series

Electronic transitions from $n_i = 3, 4, 5, \ldots$ to $n_f = 2$, give rise to spectral lines of Balmer series.

For a Balmer series line,

$$\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(2)^2} - \frac{1}{n^2}\right]$$

where, $n = 3, 4, 5, \ldots$

3. Paschen Series

Lines of this series lie in the infrared region and correspond to electronic transition from $n_i = 4, 5, 6, \ldots$ to $n_f = 3$.

$$\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(3)^2} - \frac{1}{n^2}\right]$$

where, $n = 4, 5, 6, \ldots$

4. Brackett Series

Spectral lines in the infrared region which corresponds to transition from $n_i = 5, 6, 7, \ldots$ to $n_f = 4$.

For Brackett series,

$$\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(4)^2} - \frac{1}{n^2}\right]$$

where, $n = 5, 6, 7, \ldots$

5. Pfund Series

It lies in the far infrared region of spectrum and corresponds to electronic transitions from higher orbits $n_i = 6, 7, 8, \ldots$ to orbit having $n_f = 5$.

For a spectral line in Pfund series,

$$\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(5)^2} - \frac{1}{n^2}\right]$$

where, $n = 6, 7, 8, \ldots$

NOTE
- Energy of emitted radiation,

$$\Delta E = E_2 - E_1 = RchZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\, Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

- Frequency, $v = \frac{\Delta E}{h} = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Ionisation Energy and Potential

Ionisation energy of an atom is defined as the energy required to ionise it i.e. to make the electron jump from its present orbit to infinity.

Thus, ionisation energy of hydrogen atom in the ground state $= E_\infty - E_1$

$= 0 - (-13.6 \text{ eV}) = + 13.6 \text{ eV}$

The potential through which an electron is to be accelerated so that it acquires energy equal to the ionisation energy is called the **ionisation potential**.

Therefore, ionisation potential of hydrogen atom in its ground state is 13.6V.

In general, $E_{\text{ion}} = 13.6 \frac{z^2}{n^2}$ eV or $V_{\text{ion}} = \frac{E_{\text{ion}}}{e}$

Excitation Energy and Ionisation Potential

Excitation energy is the energy required to excite an electron from a lower energy level to a higher energy level. The potential through which an electron is accelerated so as to gain requisite ionisation energy is called the **ionisation potential**.

Thus, first excitation energy of hydrogen atom

$= E_2 - E_1$

$= -3.4 - (-13.6)$ eV

$= +10.2$ eV

Similarly, second excitation energy of hydrogen atom

$= E_3 - E_1$

$= -1.51 - (-13.6)$

$= 12.09$ eV

NOTE
- Total energy of a closed system is always negative and its magnitude is the binding energy of the system.
- Kinetic energy of a particle can't be negative, while the potential energy can be zero, positive or negative.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of

(a) 1Å (b) 10^{-10}cm
(c) 10^{-12}cm (d) 10^{-15} cm

2 To explain theory of hydrogen atom, Bohr considered

(a) quantisation of linear momentum
(b) quantisation of angular momentum
(c) quantisation of angular frequency
(d) quantisation of energy

3 For the Bohr's first orbit of circumference $2\pi r$, the de-Broglie wavelength of revolving electron will be

(a) $2\pi r$ (b) πr (c) $\frac{1}{2\pi r}$ (d) $\frac{1}{4\pi r}$

4 Which of the following transitions in hydrogen atoms emit photons of highest frequency?

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$
(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

5 In the Bohr's model of the hydrogen atom, let r, V and E represents the radius of the orbit, the speed of electron and the total energy of the electron, respectively. Which of the following quantities is proportional to the quantum number n?

(a) E/v (b) r/E (c) vr (d) rE

6 The ratio of the kinetic energy to the energy of an electron in a Bohr's orbit is

(a) – 1 (b) 2
(c) 1: 2 (d) None of these

7 If the atom $_{100}Fm^{257}$ follows the Bohr's model and the radius of last orbit of $_{100}Fm^{257}$ is n times the Bohr's radius, then find the value of n ?

(a) 100 (b) 200 (c) 4 (d) 1/4

8 Taking the Bohr's radius as $a_0 = 53$ pm, the radius of Li^{2+} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm (b) 27 pm
(c) 18 pm (d) 13 pm

9 In a hypothetical Bohr's hydrogen atom, the mass of the electron is doubled. The energy E_0 and radius r_0 of the first orbit will be (a_0 is the Bohr radius)

(a) $E_0 = -27.2$ eV ; $r_0 = a_0/2$
(b) $E_0 = -27.2$eV ; $r_0 = a_0$
(c) $E_0 = -13.6$eV ; $r_0 = a_0/2$
(d) $E_0 = -13.6$eV ; $r_0 = a_0$

10 As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion

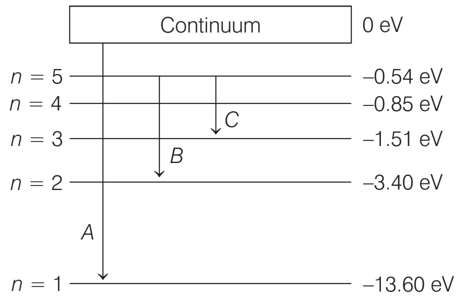
→ JEE Main 2015

(a) its kinetic energy increases but potential energy and total energy decrease
(b) kinetic energy, potential energy and total energy decrease
(c) kinetic energy decreases, potential energy increases but total energy remains same
(d) kinetic energy and total energy decrease but potential energy increases

11 An electron jumps from the 4th orbit to the 2nd orbit of hydrogen atom. Given the Rydberg's constant $R = 10^7$ cm^{-1}, the frequency (in hertz) of the emitted radiation will be

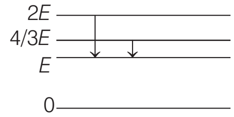
(a) $\frac{3}{16} \times 10^5$ (b) $\frac{3}{16} \times 10^{15}$ (c) $\frac{9}{16} \times 10^{15}$ (d) $\frac{3}{4} \times 10^{15}$

12 In figure, the energy levels of the hydrogen atom have been shown along with some transitions marked A, B and C. The transitions A, B and C, respectively represents



(a) the first member of the Lyman series, third member of Balmer series and second member of Paschen series
(b) the ionisation potential of H, second member of Balmer series and third member of Paschen series
(c) the series limit of Lyman series, second member of Balmer series and second member of Paschen series
(d) the series limit of Lyman series, third member of Balmer series and second member of Paschen series

13 The given diagram indicates the energy levels of a certain atom. When the system moves from $2E$ level to E, a photon of wavelength λ is emitted. The wavelength of photon produced during its transition from $4E/3$ level to E is



(a) $\lambda/3$ (b) $3\lambda/4$ (c) $4\lambda/3$ (d) 3λ

14 Energy E of a hydrogen atom with principal quantum number n is given by $E = -\frac{13.6}{n^2}$ eV. The energy of a photon ejected when the electron jumps from $n = 3$ state to $n = 2$ state of hydrogen, is approximately

(a) 1.5 eV (b) 0.85 eV (c) 3.4 eV (d) 1.9 eV

15 The ionisation potential of H-atom is 13.6 V. When it is excited from ground state by monochromatic radiations of 970.6 Å, the number of emission lines on deexcitation will be (according to Bohr's theory)

(a) 10 (b) 3 (c) 6 (d) 4

16 In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to **→ JEE Main 2013**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^{3/2}}$ (d) $\frac{1}{n^3}$

17 Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be **→ AIEEE 2012**

(a) 2 (b) 3 (c) 5 (d) 6

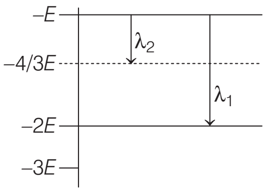
18 The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in Balmer series of single ionised helium atom is **→ AIEEE 2011**

(a) 1215 Å (b) 1640 Å (c) 2450 Å (d) 4687 Å

19 The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from **→ AIEEE 2010**

(a) $2 \to 1$ (b) $3 \to 2$ (c) $4 \to 2$ (d) $5 \to 3$

20 Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by **→ JEE Main 2017 (Offline)**



(a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$

(c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

21 Energy required for the electron excitation in Li^{2+} from the first to the third Bohr orbit is **→ AIEEE 2011**

(a) 36.3 eV (b) 108.8 eV

(c) 122.4 eV (d) 12.1 eV

22 In hydrogen atom, if the difference in the energy of the electron in $n = 2$ and $n = 3$ orbits is E, the ionisation energy of hydrogen atom is

(a) 13.2 E (b) 7.2 E

(c) 5.6 E (d) 3.2 E

23 Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is

(a) 54.4 eV (b) 13.6 eV

(c) 40.8 eV (d) 27.2 eV

Direction (Q. Nos. 24-25) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (a), (b), (c) *and* (d) *given below*

(a) Statement I is true; Statement II is true; Statement II is the correct explanation for Statement I

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(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

24 **Statement I** Bohr had to postulate that the electrons in stationary orbits around the nucleus do not radiate.

Statement II According to classical physics all moving electrons radiate electromagnetic radiation.

25 **Statement I** The different lines of emission spectra (like Lyman, Balmer, etc) of atomic hydrogen gas are produced by different atoms.

Statement II The sample of atomic hydrogen gas consists of millions of atoms.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is **→ JEE Main 2018**

(a) 25 ν_L (b) 16 ν_L (c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{25}$

2 In Rutherford's experiment, the number of alpha particles scattered through an angle of 90° is 28 per minute. Then, the number of particles scattered through an angle of 60° per minute by the same nucleus is

(a) 28 per minute (b) 112 per minute

(c) 12.5 per minute (d) 7 per minute

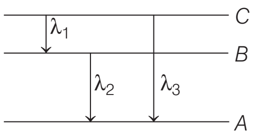
3 A hydrogen like ion having wavelength difference between first Balmer and Lyman series equal 593 Å has Z equal to

(a) 2 (b) 3 (c) 4 (d) 1

4 The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(a) 802 nm (b) 823 nm (c) 1882 nm (d) 1648 nm

5 Energy levels A, B and C of a certain atom corresponding to increasing values of energy i.e. $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following statements is correct?



(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \dfrac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

6 Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^6)^{2+}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiations are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively, then approximately which one of the following is correct?

→ JEE Main 2014

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

7 The potential energy between a proton and an electron is $\text{PE} = \dfrac{e^2}{4\pi\varepsilon_0(3R^3)}$, then the radius of the Bohr's orbit is

(a) $\dfrac{4\pi^2e^2m}{4\pi\varepsilon_0 n^2h^2}$ (b) $\dfrac{6\pi^2e^2m}{4\pi\varepsilon_0 n^3h^3}$

(c) $\dfrac{e^2m}{4\pi\varepsilon_0 h^3}$ (d) $\dfrac{2\pi e^2m}{4\pi\varepsilon_0 nh^3}$

8 A small particle of mass m moves such that potential energy $\text{PE} = \dfrac{1}{2}mr^2\omega^2$. Assuming Bohr's model of quantisation of angular momentum and circular orbit, radius of nth orbit is proportional to

(a) $\sqrt{n}$ (b) $\sqrt{n^3}$ (c) $\dfrac{1}{\sqrt{n}}$ (d) $\dfrac{1}{\sqrt{n^3}}$

9 The electric potential between a proton and an electron is given by $V = V_0 \ln\dfrac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number.

(a) $r_n \propto n$ (b) $r_n \propto \dfrac{1}{n}$

(c) $r_n \propto n^2$ (d) $r_n \propto \dfrac{1}{n^2}$

10 A hydrogen atom moves with a velocity u and makes a head on inelastic collision with another stationary H-atom. Both atoms are in ground state before collision.The minimum value of u if one of them is to be given a minimum excitation energy is

(a) 2.64×10^4 ms^{-1} (b) 6.24×10^4 ms^{-1}
(c) 2.02×10^6 ms^{-1} (d) 6.24×10^8 ms^{-1}

11 In the Bohr's model an electron moves in a circular orbit around the proton. Considering the orbiting electron to be a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in nth excited state, is

(a) $\left(\dfrac{e}{2m}\right)\dfrac{n^2h}{\pi}$ (b) $\left(\dfrac{e}{m}\right)\dfrac{nh}{2\pi}$ (c) $\left(\dfrac{e}{2m}\right)\dfrac{nh}{2\pi}$ (d) $\left(\dfrac{e}{m}\right)\dfrac{n^2h}{2\pi}$

12 A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantisation, its energy will be given by (n is an integer)

→ AIEEE 2012

(a) $\dfrac{(m_1 + m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$ (b) $\dfrac{n^2h^2}{2(m_1 + m_2)r^2}$

(c) $\dfrac{2n^2h^2}{(m_1 + m_2)r^2}$ (d) $\dfrac{(m_1 + m_2)n^2h^2}{2m_1m_2r^2}$

13 In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \dfrac{e^2}{4\pi\varepsilon_0}\left(\dfrac{1}{r^2} + \dfrac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the nth orbit in terms of the Bohr's radius $\left(a_0 = \dfrac{\varepsilon_0 h^2}{m\pi e^2}\right)$ is

→ AIEEE 2010

(a) $r_n = a_0 n - \beta$ (b) $r_n = a_0 n^2 + \beta$
(c) $r_n = a_0 n^2 - \beta$ (d) $r_n = a_0 n + \beta$

Direction (Q. Nos. 14-15) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (a), (b), (c) *and* (d) *given below*

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(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

14 **Statement I** Balmer series lies in the visible region of electromagnetic spectrum.

Statement II $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$, where $n = 3, 4, 5, \ldots\ldots$

15 **Statement I** The ionisation potential of hydrogen is found to be 13.6 eV, the ionisation potential of doubly ionised lithium is 122.4 eV.

Statement II Energy in the nth state of hydrogen atom is $E_n = -\dfrac{13.6}{n^2}$.

ANSWERS

SESSION 1	**1** (c)	**2** (b)	**3** (a)	**4** (b)	**5** (c)	**6** (a)	**7** (d)	**8** (c)	**9** (a)	**10** (a)
	11 (c)	**12** (d)	**13** (d)	**14** (d)	**15** (c)	**16** (d)	**17** (d)	**18** (a)	**19** (d)	**20** (c)
	21 (b)	**22** (b)	**23** (a)	**24** (b)	**25** (b)					
SESSION 2	**1** (d)	**2** (b)	**3** (b)	**4** (b)	**5** (b)	**6** (c)	**7** (a)	**8** (a)	**9** (a)	**10** (b)
	11 (c)	**12** (d)	**13** (c)	**14** (a)	**15** (b)					

Hints and Explanations

SESSION 1

1 Here, $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\therefore \quad 5\text{ MeV} = \frac{9\times10^9\times(2e)\times(92e)}{r}$ $\left(\because \frac{1}{2}mv^2 = 5\text{ MeV}\right)$

$\Rightarrow r = \frac{9\times10^9\times2\times92\times(1.6\times10^{-19})^2}{5\times10^6\times1.6\times10^{-19}}$

$r = 53\times10^{-14}$ m

$\approx 10^{-12}$ cm

2 While proposing his theory of hydrogen atom, Bohr considered quantisation of angular momentum as the essential condition for the stationary orbits.

3 According to Bohr's first postulate,

$mvr = \frac{nh}{2\pi}$

$\therefore \quad 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

For $n = 1$, $\lambda = 2\pi r$

4 Emission spectrum would rises when electron makes a jump from higher energy level to lower energy level. Frequency of emitted photon is proportional to change in energy of two energy levels, i.e.

$\nu = RcZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

5 According to the Bohr's postulate.

$mvr = \frac{nh}{2\pi} \Rightarrow vr = n\left(\frac{h}{2\pi m}\right)$

$\Rightarrow \quad vr \propto n$

Thus, vr is directly proportional to the principal quantum number.

6 The kinetic energy and total energy of an electron are related as, KE = $-E$

$\therefore \quad \frac{\text{KE}}{E} = -1$

7 $r_m = \left(\frac{m^2}{Z}\right)(0.53\text{ Å}) = (n\times0.53)$ Å

$\therefore \quad \frac{m^2}{Z} = n$

$m = 5$ (for $_{100}\text{fm}^{257}$ the outermost shell)

and $Z = 100$

$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$

8 On the basis of Bohr's model,

$r = \frac{n^2\lambda^2}{4\pi^2 mKZe^2} = \frac{a_0 n^2}{Z}$

Let Li^{2+} ion, $Z = 3$,

$n = 1$ for ground state.

Given, $a_0 = 53$ pm

$r = \frac{53\times(1)^2}{3}$

$\simeq 18$ pm

9 As, $r \propto \frac{1}{m}$

$\therefore \quad r_0 = \frac{1}{2}a_0$

As, $E \propto m$

$\therefore \quad E_0 = 2(-13.6)$

$= -27.2$ eV

10 As we know that, kinetic energy of an electron is KE$\propto (Z/n)^2$. When the electron makes transition from an excited state to the ground state, then n decreases and KE increases. We know that PE is lowest for ground state.

Also, TE = $-$KE. TE also decreases.

11 $\nu = \frac{c}{\lambda} = c.R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$= 3\times10^8\times10^7\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$

$= \frac{9}{16}\times10^{15}$ Hz

12 A represents series limit of Lyman series, B represents third member of Balmer series and C represents second member of Paschen series.

13 $\lambda \propto \frac{1}{\Delta E}$

$\therefore \quad \frac{\lambda'}{\lambda} = \frac{(2E-E)}{(4E/3-E)} = \frac{1}{1/3} = 3$

$\therefore \quad \lambda' = 3\lambda$

14 Given, $E_n = -\frac{13.6}{n^2}$ eV

$E_3 = -\frac{13.6}{(3)^2}$ eV $= -\frac{13.6}{9}$ eV

and $E_2 = -\frac{13.6}{(2)^2}$ eV $= -\frac{13.6}{4}$ eV

So, $\Delta E = E_3 - E_2 = -\frac{13.6}{9} - \left(-\frac{13.6}{4}\right)$

$= 1.9$ eV (approximately)

15 Using $E = \frac{hc}{\lambda} = \frac{12400}{970.6}$ eV $= 12.77$ eV

So, electron is excited upto $n = 4$

$\therefore \quad n_2 = 4$

On deexcitation number of emission lines produced $= \frac{n(n-1)}{2} = 6$

16 $\Delta E = h\nu$

$\nu = \frac{\Delta E}{h} = K\left[\frac{1}{(n-1)^2}-\frac{1}{n^2}\right]$

$= \frac{K2n}{n^2(n-1)^2} = \frac{2K}{n^3}$

17 In emission spectrum, number of bright lines is given by

$$\frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

18 $$\frac{1}{6561} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda} = 4R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R \times 4}{16}$$

$$\lambda = 1215\,\text{Å}$$

19 Infrared radiation corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$, i.e. from Paschen, Brackett and Pfund series. Thus, the transition corresponds to $5 \to 3$.

20 We have, $\lambda = \frac{hc}{\Delta E}$

$\therefore$ So, ratio of wavelengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

21 $$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\left[\because \Delta E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\right]$$

$$= 13.6(3)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 108.8\text{ eV}$$

22 $E_3 - E_2 = E$

or $$\frac{E_1}{9} - \frac{E_1}{4} = E$$

or $$E_1 = -7.2E$$

$\therefore$ Ionisation energy of hydrogen atom is $7.2\,E$.

23 Excitation energy,

$$\Delta E = E_2 - E_1 = 13.6Z^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$40.8 = 13.6Z^2 \times \frac{3}{4}$$

$\therefore$ $Z = 2$

So, required energy to remove the electron from ground state

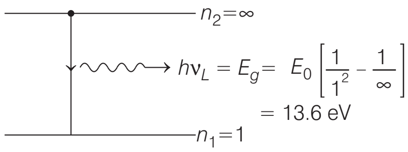
$$= +\frac{13.6Z^2}{(1)^2} = 13.6(Z)^2 = 54.4\text{ eV}$$

24 Bohr's postulated that, electron instead of revolving in any orbit around the nucleus, revolves only in some specific orbits. These orbits are called the non-radiating orbits or the stationary orbits. The electrons revolving in these orbits do not radiate any energy. They radiate only when they go from one orbit to the next lower orbit.

25 A single atom can have only one transition at time, we are observing different lines due to large number of transitions taking place simultaneously that occurred in different atoms of the sample.

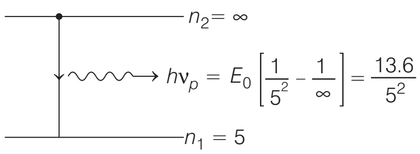
SESSION 2

1 *Series limit occurs in the transition* $n_2 = \infty$ to $n_1 = 1$ in Lyman series and $n_2 = \infty$ to $n_1 = 5$ in Pfund series. For Lyman series,



$$h\nu_L = 13.6 \quad \text{...(i)}$$

In Pfund series



$$h\nu_p = \frac{13.6}{5^2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$25h\nu_p = h\nu_L$$

$\therefore$ $$\nu_p = \frac{\nu_L}{25}$$

2 According to Rutherford's scattering formula, if the α-particles scattered at an angle θ is directly proportional to $\frac{1}{\sin^4(\theta/2)}$, then $N_\theta = \frac{K}{\sin^4(\theta/2)}$

when $\theta = 90°$,

$N_\theta = 28\text{ min}^{-1}$

$$\Rightarrow 28 = \frac{K}{\sin^4(45°)} = 4K \Rightarrow K = 7$$

Thus $$N_\theta = \frac{7}{\sin^4(\theta/2)}$$

Hence, the number of α-particles scattered at an angle of 60° per minute is $N'_\theta = \frac{7}{\sin^4 30°} = \frac{7}{(1/2)^4}$

$= 7 \times 16 = 112$ per minute

3 $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$

$$\Delta\lambda = \frac{1}{RZ^2\left(\frac{1}{4} - \frac{1}{9}\right)} - \frac{1}{RZ^2\left(\frac{1}{1} - \frac{1}{4}\right)}$$

$$593 = \frac{36}{5RZ^2} - \frac{4}{3RZ^2}$$

$$Z^2 = \frac{88}{15R\,\Delta\lambda}$$

$$= \frac{88}{15(1.097 \times 10^7)(593 \times 10^{-10})}$$

$$= 9$$

$\Rightarrow Z = 3$

4 The series in UV-region is Lyman series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$\therefore$ $$122 = \frac{\frac{1}{R}}{\frac{1}{(1)^2} - \frac{1}{(2)^2}} \quad \text{...(i)}$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$\therefore$ $$\lambda = \frac{\frac{1}{R}}{\frac{1}{(3)^2} - \frac{1}{\infty}} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\lambda = 823.5\text{ nm} \approx 823$$

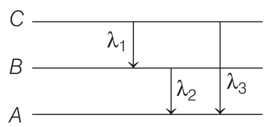
5 Let energy corresponding to state A, B and C be E_A, E_B and E_C.

So, from figure

$$(E_C - E_B) + (E_B - E_A) = (E_C - E_A)$$

or $$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$$

$\Rightarrow$ $$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$



6 As we know that,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\lambda = \frac{4}{3RZ^2}$$

$$\lambda_1 = \frac{4}{3R}, \lambda_2 = \frac{4}{3R}$$

$$\lambda_3 = \frac{4}{12R}, \quad \lambda_4 = \frac{4}{27R}$$

$\Rightarrow$ $$\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

7 $F = \frac{-dPE}{dR} = -\frac{e^2}{4\pi\varepsilon_0 R^4}$

$\Rightarrow \quad \frac{mv^2}{R} = \frac{e^2}{4\pi\varepsilon_0 R^4}$.

Also, $mvR = \frac{nh}{2\pi}$

$\therefore \quad \frac{m}{R}\left(\frac{nh}{2\pi mR}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 R^4}$

$\Rightarrow \quad R = \frac{4\pi^2 e^2 m}{4\pi\varepsilon_0 n^2 h^2}$

8 $F = \frac{-dPE}{dr} = -m\omega^2 r$.

Since, $mvr = \frac{nh}{2\pi}$

or $\quad mr^2\omega = \frac{nh}{2\pi} \quad [\because v = r\omega]$

$\Rightarrow \quad r^2 = \frac{nh}{2\pi m\omega}$

$\Rightarrow \quad r = \sqrt{\frac{nh}{2\pi m\omega}}$

$\Rightarrow \quad r \propto \sqrt{n}$

9 $U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$

and $\quad |F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$

This force will provide the necessary centripetal force. Hence,

$$\frac{mv^2}{r} = \frac{eV_0}{r}$$

or $\quad v = \sqrt{\frac{eV_0}{m}} \quad \ldots(\text{i})$

Moreover, $\quad mvr = \frac{nh}{2\pi} \quad \ldots(\text{ii})$

On dividing Eq. (ii) by Eq. (i), we have

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$$

or $\quad r_n \propto n$

10 Momentum, $mu = 2mv$

$\Rightarrow \quad v = \frac{u}{2}$

$$\Delta E = \frac{1}{2}mu^2 - \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2$$

$$= \frac{mu^2}{4}$$

$$\frac{1}{4}mu^2 = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{4}(1.0078)(1.66 \times 10^{-27})u^2$$

$$= 10.2 \times 1.6 \times 10^{-19}$$

$\Rightarrow \quad u = 6.24 \times 10^4 \text{ ms}^{-1}$

11 As, $i = \frac{e}{T}$

and magnetic moment $M = iA$

$(\because A = \pi r^2)$

$\therefore \quad M = \frac{e}{T}\cdot \pi r^2 \quad \ldots(\text{i})$

Now, $\quad T = \frac{2\pi r}{v}$

It becomes,

$$M = \frac{e\cdot \pi r^2}{2\pi r / v} = \frac{evr}{2} \quad \ldots(\text{ii})$$

Also, $\quad mvr = \frac{nh}{2\pi}$

$$vr = \frac{nh}{2\pi m}$$

Putting this value in Eq. (ii), we get

$$M = \frac{e\cdot nh}{2\cdot 2\pi m}$$

$$= \left(\frac{e}{2m}\right)\frac{nh}{2\pi}$$

12 Rotational kinetic energy of the two body system rotating about their centre of mass is

$$\text{RKE} = \frac{1}{2}\mu\omega^2 r^2,$$

$$\therefore \quad \text{RKE} = \frac{1}{2}\mu\cdot\left(\frac{nh}{2\pi\mu r^2}\right)^2 r^2$$

$$\left(\begin{array}{l}\mu = \text{reduced mass} = \dfrac{m_1 m_2}{m_1 + m_2}\\ L = \dfrac{nh}{2\pi} = \mu\omega r^2\end{array}\right)$$

$$= \frac{n^2h^2}{8\pi^2\mu r^2} = \frac{n^2 h^2}{2\mu r^2}$$

$$= \frac{(m_1 + m_2)n^2 h^2}{2m_1 m_2 r^2}$$

$$\left[\text{here, } h^2 = \frac{\lambda}{4\pi}\right]$$

13 According to question, the force between nucleus and electron provide necessary centripetal force,

$$\frac{e^2}{4\pi\varepsilon_0}\left(\frac{r+\beta}{r^3}\right) = \frac{mv^2}{r} \quad \ldots(\text{i})$$

Also, $mvr = \frac{h}{2\pi} \quad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$$\frac{e^2}{4\pi\varepsilon_0}\left(\frac{r+\beta}{r^3}\right) = \frac{mn^2h^2}{4\pi^2 m^2 r^3}$$

$\Rightarrow \quad r + \beta = \frac{\varepsilon_0 n^2 h^2}{\pi me^2}$

$\Rightarrow \quad r_n = a_0 n^2 - \beta$

$$\left(\because a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$$

14 The wavelength in Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5\ldots$$

$$\frac{1}{\lambda_{\text{max}}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_{\text{max}} = \frac{36}{5R}$$

$$\lambda_{\text{max}} = \frac{36 \times 1}{5 \times 1.097 \times 10^7} = 6563 \text{ Å}$$

and $\quad \frac{1}{\lambda_{\text{min}}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$$\lambda_{\text{min}} = \frac{4}{R} = \frac{4}{1.097 \times 10^7}$$

$$= 3646 \text{ Å}$$

The wavelengths 6563 Å and 3646 Å lie in visible region. Therefore, Balmer series lies in visible region.

15 From Bohr's theory, the energy of hydrogen atom in the nth state is given by $E_n = -\frac{13.6}{n^2}$ eV. For an atom of atomic number Z, with one electron in the outer orbit (singly ionised He or doubly lithium) we use. $E_n = -\frac{13.6 Z^2}{n^2}$ eV,

where Z is the atomic number. Hence, ground state energy of doubly ionised lithium is $\frac{-13.6 \times 9}{(1)^2} = -122.4$ eV

Ionisation potential (potential to be applied to electron to overcome this energy) is 122.4 V.